

Combinatorial Optimization with Rydberg Atoms: the Barrier of Interpretability

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1) The MWIS problem

- **Maximum Weighted Independent Set:** given a weighted graph, amounts to find a set of independent (non-adjacent) vertices with maximal weight
- Unless $P = NP$, impossible to solve, even approximately, in polynomial time
- Useful problem with industrial applications

2) Unitary-disk (UD) graphs

- Rydberg atoms natively encode MWIS^[1]:

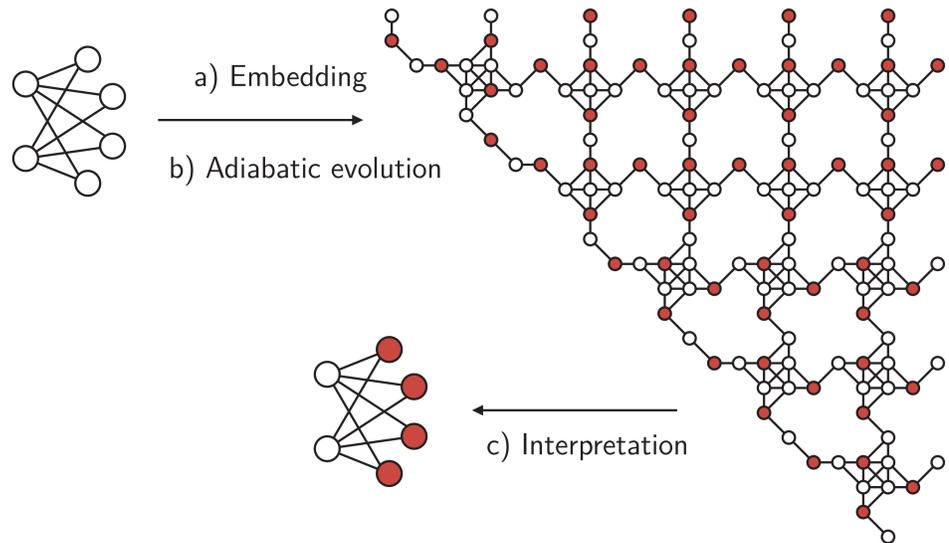
$$H = - \sum_i \delta_i n_i + U \sum_{i \sim j} n_i n_j$$

with $U \gg \delta_i > 0$ and $n_i \in \{0,1\}$ indicates if vertex i is selected or not

- In 2D this only describes the class of UD graphs, i.e. with edges between vertices separated by less than a unit distance
- Embedding schemes are required to map non-UD MWIS instances onto UD-MWIS

3) Embedding MWIS into UD-MWIS with Rydberg atoms

- An example of embedding scheme relies on crossing lattice graphs^[2]:



- Issue: imperfect annealing lead to non-interpretable configurations with domain-wall-like defects:

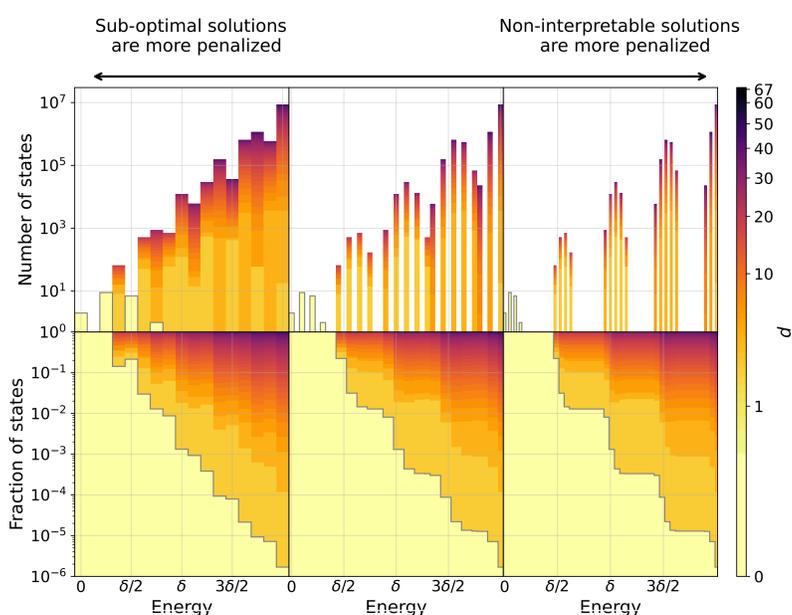


- Approach: we study the Hamming distance d to the nearest interpretable configuration

4) Exponential decay of interpretability in embedding graphs^[3]

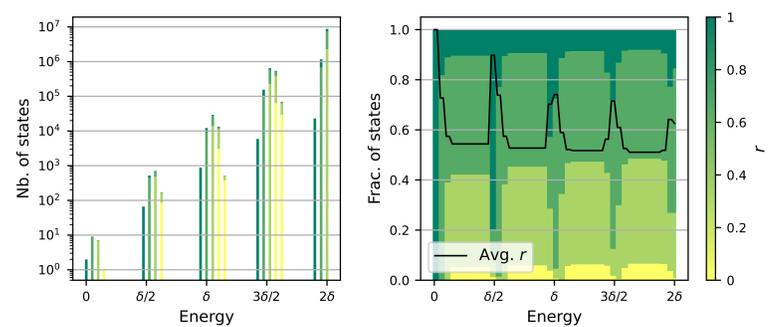
a) Low-energy DoS in a random embedding graph

- Several varying parameters: graph size & energy penalties
- Consistent behavior: exp. decay of the fraction of low d values in increasing energy windows (*bottom panels*)



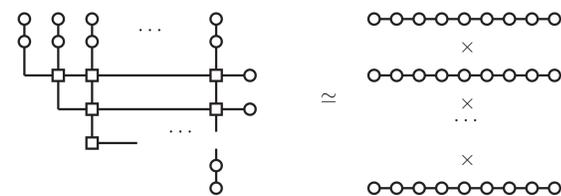
b) Approximation ratio

- Link between the approximation ratio r and the energy window



c) Approximation by simpler graphs

- Products of paths graphs reproduce the distribution of the distance d



5) Conclusions

Two very different regimes of energies are identified:

- For $E \leq \delta/2$, measurements outputs are easy to interpret as solutions of the initial problem
- For $E \geq \delta/2$, post-processing is exponentially hard when E increases

In practice, defects due to finite annealing time jeopardize approximate MWIS optimization with embedding schemes. Our approach is expected to yield similar results for a broad class of embedding techniques tailored for UD-MWIS optimization, integer factorization, QUBO, etc.

6) References

- [1] H. Pichler et al., arXiv:1808.10816 (2018)
- [2] M.-T. Nguyen et al., PRXQ 4, 010 16 (2023)
- [3] C. de Correc, T. Ayrat & C. Bertrand, coming soon on ArXiv!